Fluid Mechanics (ME 201)

Instructor: Dr. Sudhakar Yogaraj

School of Mechanical Sciences, IIT Goa.

Tutorial 6 – Governing equations in differential form

1. Do the following velocity fields represent possible incompressible fluid flow?

   (a) $u = x + y + z^2, \quad v = x - y + z, \quad w = 2xy + y^2 + 4$

   (b) $u = xyzt, \quad v = -xyzt^2, \quad w = z^2(xt^2 - yt)/2$

   (c) $u = y^2 + 2xz, \quad v = -2yz + x^2yz, \quad w = \frac{1}{2}x^2z^2 + x^3y^4$

2. An idealized incompressible flow has the proposed three-dimensional velocity distribution

   $$\mathbf{U} = 4xy^2\mathbf{i} + f(y)\mathbf{j} - zy^2\mathbf{k}$$

   Find appropriate form of function $f(y)$. Answer: $f(y) = -y^3$

3. Write the continuity equation

   $$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla} \cdot (\rho \mathbf{u}) = 0$$

   in terms of volumetric strain rate. From this, write the mass conservation equation for an incompressible flow.

4. A two-dimensional boundary layer over a flat plate can be approximated by

   $$\frac{u}{U} \approx \left(2\frac{y}{\delta} - \frac{y^2}{\delta^2}\right) \quad \text{for} \quad y \leq \delta$$

   where $\delta = C\sqrt{x}$, and C is a constant. Find an expression for wall-normal velocity $v(x, y)$ for $y \leq \delta$ (ii) Plot u and v against y. Answer: $\frac{v}{U} = \frac{\delta}{x}\left[\frac{1}{2}\left(\frac{y}{\delta}\right)^2 - \frac{1}{3}\left(\frac{y}{\delta}\right)^3\right]$

5. Consider a steady fully developed laminar flow of an incompressible fluid between two infinite parallel plate. Derive expressions for velocity field, average velocity, and skin-friction coefficient when both plates are held stationary. Take $x-$axis along the bottom plate. [This is called Plane Poiseuille flow] Answer:$u = \frac{1}{2\mu}\frac{dp}{dx}(y^2 - Hy)$

6. Water at 20^{o}C is flowing between a two-dimensional channel in which the top and bottom walls are 1.5 mm apart. If the average velocity is 2 m/s, find out (i) maximum velocity, (ii) pressure drop, and (iii) wall shear stress. Viscosity of water, μ=0.00101 kg/ms.
   . Answer: (i) 3 m/s (ii) -10773.3 N/m^3 (iii) 8.08 N/m^2

7. Water at 60^{o}C flows between two large flat plates. The lower plate moves to the left at a speed of 0.3 m/s. The plate spacing is 3 mm. Assuming laminar flow, determine the pressure gradient required to produce zero net flow through the cross section. ($\mu = 4.7 \times 10^{-4}$ kg/ms at 60^{o}C) Answer: -94 N/m^3

8. Consider steady, incompressible, parallel, laminar flow of a film of oil falling slowly down an infinite vertical wall. The oil film thickness is in $x-$direction is h. There is no applied pressure driving the flow i.e., the oil falls by gravity alone. Calculate the velocity and pressure fields in the oil film and sketch the normalized velocity profile. You may neglect changes in the hydrostatic pressure of the surrounding air. Answer: $v = \frac{\rho g x}{2\mu}(x - 2h)$

9. Prove that an incompressible Newtonian fluid obeys the relation $\nabla \cdot \boldsymbol{\tau} = \mu \nabla^2 \mathbf{U}$, where $\boldsymbol{\tau}$ is deviatoric stress tensor and **U** is velocity vector.

10. A two-dimensional converging duct is being designed for a high-speed wind tunnel. The bottom wall of the duct is to be flat and horizontal, and the top wall is to be curved in such a way that the axial wind speed u increases approximately linearly from u_1 =100 m/s at section (1) to u_2 = 300 m/s at section (2). Meanwhile, the air density ρ is to decrease linearly from ρ_1=1.2 kg/m^3 at section (1) to ρ_1=0.85 kg/m^3 at section (2). The converging duct is 2 m long and is 2 m high at section (1). Predict the y-component of velocity, v(x, y), in the duct.