Fluid Mechanics (ME 201)

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Tutorial 5 – Dimensional analysis

1. Write dimensions of the following in terms of primary dimensions: surface tension (σ), dynamic viscosity (μ), kinematic viscosity (ν), strain rate ($\dot{\epsilon}$)

2. Check dimensional homogeneity of the following equations

$$\tau = \rho\nu\frac{du}{dy}$$

$$H = \frac{4\sigma\ \cos\theta}{\rho g D}$$

$$p + \frac{1}{2}\rho U^2 + \rho g h = P_0$$

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}$$

$$a = \sqrt{\gamma R T}$$

3. Drag force on a smooth sphere depends on flow velocity, sphere diameter, fluid density and viscosity. Obtain dimensionless groups that can be used to correlate experimental data.
   Answer: $\frac{F}{\rho U^2 D^2} = f\left(\frac{\rho U D}{\mu}\right)$

4. At very low speeds, in the above problem, drag is independent of fluid density. At high speeds it is independent of viscosity. Using dimensional analysis, determine how the drag force depends on its velocity for these two cases.
   Answer: $F \propto U^2$ and $F \propto U$ for high and low speeds respectively

5. Lift force of a high speed aircraft depends on velocity of the flight U, the characteristic geometrical dimension of the aircraft L, density ρ, viscosity μ, and isentropic bulk modulus of elasticity E_s. Find independent dimensionless quantities which describe the lift force.
   Answer: $\frac{F}{\rho U^2 D^2} = g\left(\frac{\rho U D}{\mu}, \frac{U^2}{a^2}\right)$. We can also write this as, $C_D = g(Re, M)$

6. When a small tube is dipped into a pool of liquid, surface tension causes a meniscus to form at the free surface, which is elevated or depressed depending on the contact angle at the liquid-solid-gas interface. Experiments indicate that the magnitude of capillary rise Δh, is a function of tube diameter D, liquid specific weight γ, and surface tension σ. Determine the non-dimensional Π parameters.
   Answer: $\frac{\Delta h}{D} = f\left(\frac{\sigma}{\gamma D^2}\right)$

7. The pressure drop Δp, for steady incompressible flow through a straight horizontal pipe depends on the pipe length l, average velocity U, fluid viscosity μ, pipe diameter D, fluid

density ρ, and the average roughness height ϵ. Determine a set of dimensionless group that can be used to correlate data.
Answer: $\frac{\Delta p}{\rho U^2 D^2} = f\left(\frac{\rho U D}{\mu}, \frac{l}{D}, \frac{\epsilon}{D}\right)$

8. Pressure increase (Δp) produced by a centrifugal compressor is a function of impeller diameter D, its rotation rate Ω, volume flow rate Q, and the fluid density ρ. Determine all non-dimensional Π parameters.
Answer: $\frac{\Delta p}{\rho d^2 \Omega^2} = f\left(\frac{Q}{d^3 \Omega}, Re\right)$, where $Re = \frac{\Omega d^2 \rho}{\mu}$

9. A propeller is placed in a tank of chemicals to mix them together. The diameter is D, the rotation speed is Ω, and the power to run the propeller is P. The fluid density is ρ and μ. Tests in water (ρ=1000 kg/m^3, $\mu = 10^{-3}$ kg/ms) show that a propeller D=225 mm rotating at 23 rev/s requires a driving power of 159 Watts. Calculate the speed and torque required to drive a dynamically similar propeller 675 mm in diameter in air (ρ_a=1.2 kg/m^3, $\mu = 10^{-5}$ kg/ms). Answer: $\Omega_p = 21.29$ rev/s, $T_p = 1.7273$ Nm.

10. A periodic Karman vortex street is formed when a uniform stream flows over a circular cylinder. Generate a dimensionless relationship for Karman vortex shedding frequency f as a function of the flow velocity U , fluid density ρ, fluid viscosity μ, and cylinder diameter D.
Answer: $\frac{fD}{U} = f\left(\frac{\rho U D}{\mu}\right)$, or equivalently $St = f(Re)$

11. Repeat the above problem by including speed of sound a as also a parameter affecting f.
Answer: $\frac{fD}{U} = f\left(\frac{\rho U D}{\mu}, U/a\right)$, or equivalently $St = f(Re, M)$

12. The speed of propagation U of a capillary wave in deep water depends on density ρ of water, wavelength λ, and surface tension coefficient σ. (i) Using Buckingham Pi theorem, establish relationship of U with other parameters. (ii) For a given surface tension and wavelength, how does the propagation speed changes if density is halved?
Answer: (i) $\frac{\rho U^2 \lambda}{\sigma} = C$, (ii) Increased by a factor of $\sqrt{2}$

13. A 1/6 model automobile is tested in a wind tunnel with same air properties as the prototype. The prototype automobile runs on the roads at a velocity of 60 km/hr. For dynamically similar conditions, the drag measured on the model is 500 N. Determine the drag of the prototype and power required to overcome this drag. Answer: 500 N and 8.33 kW

14. Calculate the thrust required to run a motor boat 5 m long at 100 m/s in a lake if the force required to tow a 1:30 model in a reservoir is 5 N. Neglect the viscous resistance due to water in comparison to the wave-making resistance. Answer: 135 kN

15. An aircraft is to fly at a height of 9 km (where T=-45^{o}C and p=30.2 kPa) at 400 m/s. A 1/20th scale model is tested in a pressurized wind-tunnel in which the air is at 15^{o}C. For complete dynamic similarity what pressure and velocity should be used in the wind tunnel? For air $\mu \propto \frac{T^{3/2}}{T+117}, p = \rho R T$, where T is in Kelvin, and R=287 J/kgK.
Answer: 820.95 kPa and 449.56 m/s