Fluid Mechanics (ME 201)

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Tutorial 3 – Fluid kinematics

1. For the velocity field $\overline{U} = (10x^2y\hat{i} + 15xy\hat{j} + (25t - 3xy)\hat{k})$m/s, find the acceleration of fluid particle at (1,2,-1) and at $t = 0.5$.

2. Given a temperature field $T = (xy + z + 3t)\ K$, and velocity field $\overline{U} = (xy\hat{i} + z\hat{j} + 5t\hat{k})$m/s. Compute rate of change of temperature of a particle at (1,1,1) at t=2seconds.

3. For the velocity field $\overline{U} = by\hat{i} - ax\hat{j}$ where a and b are constants, find the equation of streamline passing through the point $\left(1/\sqrt{2a}, 1/\sqrt{2b}\right)$.

4. Show that the velocity field given by $\overline{U} = (a + by - cz)\hat{i} + (d - bx + ez)\hat{j} + (f + cx - ey)\hat{k}$ of a fluid represents a rigid body motion.

5. Compute strain rate tensor and vorticity components for the velocity field $\overline{U} = (5x + 7y - 2z)\hat{i} + (8x - y + 3z)\hat{j} + (3x + 4y + 7z)\hat{k}$ at a point (1,2,-2).

6. Is the flow field given by the following velocity vector irrotational?

$$u = 2x^2 + 3y$$
$$v = -2xy + 3y^2 + 3yz$$
$$w = 1.5z^2 + 2xz - 9y^2z$$

7. Water flows through a diffuser at an increasing flowrate so that the velocity along the centerline is given by $V = u\hat{\mathbf{i}} = V_0\left(1 - e^{-ct}\right)(1 - x/l)\,\hat{\mathbf{i}}$, where V_0, c, and l are constants. Determine the acceleration associated with this flow. If $V_0 = 10$ m/s and $l = 5$ m, what value of c (other than 0) is needed to make the acceleration zero for any x at $t = 1$ s? Explain how the acceleration can be zero if the flowrate is increasing with time. [c=0.49 s^{-1}]

8. An incompressible fluid flows steadily through a parallel-plate channel of height $2h$. The fluid is entering the channel with a uniform velocity U_a. The velocity distribution at a section downstream is $u = u_{max}\left[1 - \left(\frac{y}{h}\right)^2\right]$, where u_{max} is the maximum velocity. Express u_{max} in terms of U_a. $[u_{max} = 3U_a/2]$

9. A rigid tank of volume V contains air at an absolute pressure of P and temperature T. At $t = 0$, air begins escaping through a valve with the flow area A_1. The air passing through the valve has a speed of U_1 and density ρ_1. Determine the instantaneous rate of change of density in the tank at $t = 0$ assuming it to be uniform within the tank. $\left[\frac{\partial\rho}{\partial t} = -\frac{\rho_1 U_1 A_1}{V}\right]$

10. The fluid in direct contact with a stationary solid boundary has zero velocity; there is no slip at the boundary. Thus the flow over a flat plate adheres to the plate surface and forms a boundary layer, as depicted below. The flow ahead of the plate is uniform with velocity $U = 30$ m/s. The velocity distribution within the boundary layer $(0 \leq y \leq \delta)$ along cd is approximated as $u/U = 2(y/\delta) - (y/\delta)^2$. The boundary-layer thickness at location d is $\delta = 5$ mm. The fluid is air with density $\rho = 1.24$kg/m^3. Assuming the plate width perpendicular to the paper to be $w = 0.6$ m, calculate the mass flow rate across surface bc of control volume abcd. [0.0372 kg/s]